Imperfect financial contracting and macroeconomic stability

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Abstract

This paper studies the implications of imperfect financial contracts for macroeconomic stability in the context of a stochastic dynamic general equilibrium model. We find that the equilibrium growth path might be indeterminate in an economy with financing frictions even if the aggregate production function exhibits constant returns to scale. Self-fulfilling expectations about the future price of capital lead to macroeconomic fluctuations in this economy. Impulse response analysis shows that while consumption and employment are highly procyclical, investment and the market price of capital are predicted to be negatively correlated with output.

Keywords: local indeterminacy, endogenous fluctuations,
imperfect financial markets

1 Introduction

The initial focus of real business cycle theory on short and medium-run fluctuations in productivity growth rates and technology adoption has drawn away a lot of attention in economic research from financial factors and spending habits which were traditionally viewed to restrict aggregate output in the short to medium run. In contrast to these methodological developments, the well-established co-movements of monetary and real aggregates (see Friedman (1958) and Stock and Watson (1989)) suggest that at least at business cycle frequencies financing frictions might be a source of fluctuations in economic growth. The evidence in that respect has not changed much over the last forty years and as a consequence, various authors have proposed dynamic general equilibrium models of the business cycle incorporating some kind of financial market imperfection (see Bernanke, Gertler and Gilchrist (1999), Kiyotaki and Moore (1997) or Carlstrom and Fuerst (1997)). This paper explores the consequences of relaxing the assumption of perfect financial markets for the local stability properties of equilibrium paths. This is an important theoretical issue because if the equilibrium of the economy is characterized by local indeterminacy expectational errors provide a source of shocks which might lead to persistent fluctuations in many aggregate time series such as output, employment and stock market wealth (see Benhabib and Farmer (1996) for an excellent survey of the literature on locally indeterminate equilibrium). Also, the existence of financial market imperfections implies that the first fundamental theorem of welfare cannot be applied to these models and it is therefore no longer ensured that a unique, pareto-efficient equilibrium exists. Fluctuations due to expectational errors are independent of the underlying technological shocks and therefore lead to excessive volatility of the macroeconomy and a welfare loss for indivduals who prefer to intertemporally smooth consumption. We find that the equilibrium paths of economies with imperfect financial markets might indeed be characterized by local indeterminacy even if the aggregate production function displays constant returns to scale.

Contrary to other models in the indeterminacy literature which rely on self-fulfilling expectations about future consumption, fluctuations here originate from self-fulfilling expectations about the future price of capital. The mechanism can briefly be described as follows. Agents expecting a high price of capital in the next period increase their investment expenditures today. With constant returns to scale and imperfect financial markets this investment increase triggers two effects: agents consume less and monitoring costs associated with investment increase. Indeterminacy arises if together with consumption also employment decreases which dampens the increase in the capital stock associated with the increase in investment. As a result of the employment decrease, the marginal product of capital decreases and the price of capital rises as expected, but returns to its steady-state level relatively quickly. Despite the short-lived deviation of the price of capital from its steady-styate value, the model generates persistent effects on output and employment due to the persistent effects on entrepreneur's net worth which mitigates the financing frictions in the future. Investment and the capital stock remain high for many periods and also the changes in consumption and employment are quite persistent. The model therefore provides a financial propagation mechanism that is completely absent from business cycle models which rely on technology shocks.

In the next section, we will describe the model derive the resulting equilibrium conditions. In section 3 we parametrize the utility function of the representative agent and characterize numerically and graphically the parameter space in which the equilibrium of the economy is indeterminate. We also compute impulse response functions for an economy with indeterminate equilibrium and compare them to the standard empirical results. Section 4 concludes and suggests directions for future research.

2 The Model Setup

We study the stability properties of the model introduced by Carlstrom and Fuerst (1997). There is a continuum of two types of infinitely-lived agents, referred to as workers and entrepreneurs, with mass normalized to 1. Only entrepreneurs have access to the capital-production technology and can obtain financing through a competitive sector of financial intermediaries that operates without cost. The risks associated with any individual business are diversified away by the financial intermediaries and workers therefore obtain a certain rate of return on their savings. Let me begin with describing the economic problem of workers. [1]

2.1 Workers

The fraction of workers in the population is $(1-\eta)$. They are assumed to be risk-averse and maximize

$$E_0\left[\sum_{t=0}^{\infty}\beta^t U\left(c_t, 1-h_t\right)\right]$$

with respect to c_t and h_t and subject to their budget constraint. Here E_t denotes the expectation conditional on information available at time t $\beta \in (0,1)$ is the discount rate of workers, c_t is consumption, h_t is the amount

[1] Lowercase letters denote individual quantities and uppercase letters denote aggregate quantities, while greek letters denote parameters.

of hours worked and the time endowment is normalized to 1. The budget constraint is

$$c_t = W_t h_t + R_t k_t - Q_t(k_{t+1} - (1-\delta)\,k_t)$$

where W_t, R_t and k_t are the wage rate, the return to capital and the capital stock of each worker, respectively. δ is the depreciation rate and Q_t denotes the price of capital goods in terms of consumption goods. The optimal choices of workers are described by the labour supply curve

$$-U_h(t)/U_c\,(t) = W_t \tag{1}$$

and the consumption Euler equation

$$Q_t U_c(t) = \beta E_t \left\{U_c(t+1)\left[Q_{t+1}(1-\delta) + R_{t+1}\right]\right\} \tag{2}$$

In each period, aggregate output of the consumption good in this economy, denoted by Y_t, is produced competitively with a constant returns to scale technology using aggregate capital K_t (which in slight abuse of notational conventions includes both the capital of workers and that of entrepreneurs), aggregate labour and aggregate entrepreneurial skills, which are denoted by HE_t.

$$Y_t = F(K_t, H_t, HE_t)$$

The price of the consumption good is normalized to 1 and competitive factor markets ensure that the three factors of production receive their respective marginal products: $R_t = F_1(t)$, $W_t = F_2(t)$, $X_t = F_3(t)$. X_t represents the wage rate for entrepreneurial skills. The assumption of entrepreneurial labour income is necessary in order to ensure positive net wealth of entrepreneurs in each period, which is required in order for the financial contracting problem described below to be well defined.

2.2 The debt contract

Before describing the behavior of entrepreneurs, we will now turn to the derivation of the optimal debt contract. Debt is welfare-improving in this economy, because only entrepreneurs can produce the capital good, but may not have enough current wealth to purchase the optimal quantity of factor inputs. The presence of asymmetric information is the reason why access to the financial market is restricted for entrepreneurs. As mentioned before, we assume the existence of financial intermediaries, which collect the savings of workers and then lend to many entrepreneurs simultaneously. In this way the financial intermediaries are able to diversify away the risks associated with any individual debt contract and guarantee a certain rate of return to the workers[2]. The most important restriction that is used in order to simplify the implementation of the contracting problem into the dynamic general equilibrium model is that only intraperiod contracts are feasible and that there is no stochastic monitoring. Also there is no possibility of tracking the behavior of entrepreneurs over time.

Optimal debt contracts are written between the financial intermediaries and the entrepreneurs. Both parties are risk-neutral and entrepreneurs have positive net wealth n (in this part we drop the time indices, since there are no dynamic considerations). The capital production technology operated by each entrepreneur is linear and allows them to convert 1 consumption good into ω units of capital. ω is an idiosyncratic random variable with non-negative support, a mean of 1, some density function $\phi(\omega)$ and a cumulative distribution function $\Phi(\omega)$. It is assumed to be i.i.d. across time and across entrepreneurs. The contracting problem arises, because only the entrepreneur operating the technology can observe the realization of ω costlessly. Other agents must pay some monitoring cost μi, which is proportional to the size of the project i, in order to learn this value. This asymmetry of information creates a moral hazard problem, because entrepreneurs have an incentive to report a low value of ω. By the revelation principle however, the optimal contract will be structured such that the entrepreneurs always truthfully report ω.

In the absence of the possibility of stochastic monitoring and with all economic rents flowing to the entrepreneur (that means free entry into lending), the optimal contract maximizes the expected return to investing of entrepreneurs subject to the participation constraints of entrepreneurs and the no-profit-condition for financial intermediaries. Being the residual claimant, the expected return of the entrepreneur is given by the difference between the expected output of the project and the expected cost of finance:

$$Q\left[\int_{\bar{\omega}}^{\infty}\omega i\phi(\omega)\,d\omega-(1-\Phi(\bar{\omega}))\left(1+r^{k}\right)(i-n)\right]$$

where the expected return is measured in terms of consumption goods, r^k is the interest rate specified by the optimal contract and $(i-n)$ is the amount borrowed by the individual entrepreneur. Because of truthful reporting, monitoring and default will occur exactly when the entrepreneur is unable to repay the debt, i.e. if

$$\omega<\frac{\left(1-r^{k}\right)(i-n)}{i}\equiv\bar{\omega}$$

[2] In fact, because we assume that there is free-entry into lending and the loans are intraperiod loans the rate of return will be exactly 0.

Effectively, the optimal contract specifies a pair of decision variables $(i, \bar{\omega})$, where $\bar{\omega}$ is the cutoff level of the reported idiosyncratic shock below which monitoring (and default) will occur. The price of capital Q and the net wealth of entrepreneurs n are taken as parameters and determine the specific values of $(i, \bar{\omega})$ for each contract.

Substituting the definition of $\bar{\omega}$ into the objective function above yields

$$Qi\left[\int_{\bar{\omega}}^{\infty} \omega\phi\left(\omega\right)d\omega - \left(1-\Phi\left(\bar{\omega}\right)\right)\bar{\omega}\right] \equiv Qif(\bar{\omega})$$

where $f(\bar{\omega}) = \left[\int_{\bar{\omega}}^{\infty} \omega\phi\left(\omega\right)d\omega - \left(1-\Phi\left(\bar{\omega}\right)\right)\bar{\omega}\right]$ can be interpreted as the fraction of expected net capital output received by the entrepreneur. Since the loans are intraperiod loans, the maximization is subject to the intermediaries and the entrepreneurs not receiving a negative return in expectation,

$$\begin{aligned} & Q\left[\int_{0}^{\bar{\omega}} \omega i\phi\left(\omega\right)d\omega - \Phi\left(\bar{\omega}\right)\mu i + \left(1-\Phi\left(\bar{\omega}\right)\right)\left(1+r^{k}\right)(i-n)\right] \\ \equiv\ & Qig(\bar{\omega}) \geq (i-n) \end{aligned}$$

and

$$Qif(\bar{\omega}) \geq n$$

Analogously to $f(\bar{\omega})$, $g(\bar{\omega}) = \left[\int_{0}^{\bar{\omega}} \omega i\phi\left(\omega\right)d\omega - \Phi\left(\bar{\omega}\right)\mu i + \left(1-\Phi\left(\bar{\omega}\right)\right)\bar{\omega}\right]$ can be interpreted as the fraction of expected net capital output received by the financial intermediaries and adding the two terms yields

$$f(\bar{\omega}) + g(\bar{\omega}) = 1 - \Phi\left(\bar{\omega}\right)\mu$$

which shows that a fraction $\Phi\left(\bar{\omega}\right)\mu$ of the produced capital is destroyed by monitoring and the remaining capital is divided between the entrepreneurs and the financial intermediaries. The relevant first-order conditions pinning down the optimal values of $\bar{\omega}$ and i in each period as functions of Q and n are

$$Q\left\{1-\Phi\left(\bar{\omega}\right)\mu + \phi\left(\bar{\omega}\right)\mu\left[\frac{f\left(\bar{\omega}\right)}{f'\left(\bar{\omega}\right)}\right]\right\} = 1 \tag{3}$$

$$i = \frac{n}{1-Qg\left(\bar{\omega}\right)} \tag{4}$$

In the following, it will be important to note that the optimal cutoff level $\bar{\omega}$ is the same for all the entrepreneurs, regardless of their current net worth and only depends on the price of capital Q. This is due to the linear technology of entrepreneurs, which equalizes their marginal productivity regardless of the size of operations and the sources of finance.

2.3 Entrepreneurs

Entrepreneurs are assumed to be risk-neutral, but discount the future more heavily than workers do. This assumption is contrary to the "received wisdom" that entrepreneurs are more patient than other people and therefore willing to accumulate more capital. In the presence of financial market imperfections this statement must be qualified. Since with imperfect financial markets there is a positive wedge between the expected rates of return of lenders and entrepreneurs, entrepreneurs do not save because they are more patient, but rather because they can get a higher return. In fact, what is needed in order to prevent them from saving too heavily in initial periods and become completely self-financing, is that entrepreneurs are less patient than other agents. This is precisely what is assumed here and we chose the spread between the discount rates of entrepreneurs and workers such that it neutralizes the expected internal rate of return in the steady-state. Specifically, entrepreneurs maximize

$$E_0 \left[\sum_{t=0}^{\infty} (\beta\gamma)^t ce_t \right]$$

where ce_t denotes consumption by entrepreneurs at time and $\gamma \in (0,1)$ is the additional rate of discounting and defined by

$$\gamma = \frac{1}{r^E}$$

where r^E denotes the expected internal rate of return in steady state to be defined below. Entrepreneurs supply inelastically one unit of entrepreneurial skills and accumulate their own capital holdings, denoted by z_t at time t. Their budget constraint is

$$ce_t = n_t r_t^E - z_{t+1} Q_t$$

where n_t is the individual entrepreneur's net wealth and r_t^E the individual expected rate of return on internal funds in the production of new capital goods at time t. From the optimal contracting problem above, this expected internal rate of return is

$$r_t^E = \frac{Q_t i_t f(\bar{\omega}_t)}{n_t} = Q_t \left[\frac{f(\bar{\omega}_t)}{1 - Q_t g(\bar{\omega}_t)} \right] > 1$$

Because this expected internal rate of return is greater than 1, the risk-neutral entrepreneurs will always pour their entire net worth into the loan contract to be negotiated. Net worth at time t in terms of the consumption goods is given by the sum of entrepreneurial wages, rental income from the capital stock and the value of the depreciated capital stock

$$n_t = X_t + R_t z_t + Q_t z_t (1 - \delta) \tag{5}$$

If optimal entrepreneurial consumption is always positive, the optimal consumption decisions are captured by the following Euler equation

$$Q_t = \beta\gamma E_t \left\{ \left[Q_{t+1}(1-\delta) + R_{t+1} \right] . Q_{t+1} \left[\frac{f(\bar{\omega}_{t+1})}{1 - Q_{t+1} g(\bar{\omega}_{t+1})} \right] \right\} \tag{6}$$

It should be noted that, because of the linearity of utility, no individual variables appear in this equation (if we recall that the optimal cutoff level $\bar{\omega}$ only depends on the price of capital Q) and hence the dispersion in entrepreneurial net worth that is caused by the idiosyncratic productivity shocks does not matter for the entrepreneur's consumption-saving decisions. All solvent entrepreneurs will consume according to this Euler equation and those that had to declare bankruptcy at date t loose all their wealth, consume zero consumption goods in period t, and have to start from scratch in the next period. The linearity of utility in consumption makes entrepreneurs indifferent to these risks. In the definition of equilibrium below, we will further use the aggregated budget constraint of entrepreneurs, which is given by

$$\begin{aligned} Z_{t+1} &= \frac{\eta(n_t r_t^E - ce_t)}{Q_t} \\ &= \left\{ \eta X_t + Z_t \left[Q_t(1-\delta) + R_t \right] \right\} \left[\frac{f(\bar{\omega}_t)}{1 - Q_t g(\bar{\omega}_t)} \right] - \frac{\eta ce_t}{Q_t} \end{aligned} \tag{7}$$

2.4 Equilibrium

The model is closed by the market clearing conditions for the four markets in the economy, the two labour markets, the consumption good market and the capital market, respectively

$$H_t = (1-\eta) h_t$$

$$HE_t = \eta$$

$$(1-\eta)c_t + \eta ce_t + \eta i_t = Y_t \tag{8}$$

$$K_{t+1} = (1-\delta)K_t + I_t \tag{9}$$

where I_t stands for aggregate realized investment and is equal to total production of new capital goods net of monitoring costs

$$I_t = \eta i_t (1 - \Phi(\bar{\omega}_t)\mu)$$

A recursive competitive equilibrium in this economy is defined by decision rules for $C_{t+1}, Q_{t+1}, K_{t+1}, Z_{t+1}, CE_t, H_t, I_t, N_t, \bar{\omega}_t$, which are stationary functions of the state variables (C_t, K_t, Q_t, Z_t) and satisfy equations (1)-(9).

3 Detecting indeterminate equilibria

As mentioned before, the purpose of this paper is to examine the robustness of the property of determinacy of equilibrium in SDGE-models with imperfect financial markets. From the example of Carlstrom and Fuerst (1997), we know that at least for one specific set of calibrated parameters the present model possesses a determinate equilibrium. The question, we are asking in this section is how robust this property of determinate equilibrium is to changes in the calibration. In other words, we want to see whether there are other plausible calibrations of the same model, for which the model possesses an indeterminate equilibrium. Before going on to do that, we briefly want to review the requirements for and the interpretation of indeterminacy and the main empirical problems these models face[3].

Put simply, the property of indeterminacy means that the model possesses infinitely many equilibrium paths arbitrarily close to each other. This implies that a problem of equilibrium selection arises. Technically, indeterminacy of equilibrium corresponds to „excess stability" of the underlying system of difference (or differential) equations. In determinate models, the number of stable roots equals the number of predetermined variables (variables whose next period value is completely determined by the current state variables). If this is the case, there are exactly enough initial conditions to pin down a unique equilibrium path (the saddle path). In models with indeterminate equilibrium, the number of stable roots of the system is larger than the number of predetermined variables. It follows that one or more initial conditions are missing and many equilibrium paths (i.e. sequences of the state variables which satisfy the system of difference equations derived from the equilibrium conditions) exist. In the business cycle literature this property is exploited by letting expectational errors generate fluctuations of real variables in the model. Since these fluctuations are driven by expectational errors, they do not have a relation to economic fundamentals and generally produce non-pareto-optimal allocations if agents are risk-averse. The possibility of indeterminate equilibrium has been known at least since Blanchard and Kahn (1980) but only recently some economists have taken this possibility seriously and showed that for perfectly plausible parameter values SDGE-models of the business cycle may display indeterminacy (see Benhabib and Farmer (1996), Schmitt-Grohe (2000)).

The methodology used in the business cycle literature is by now very well-known. One first solves for the steady-state of the underlying growth model and then log-linearizes the equilibrium conditions around the steady-state. In the present case, we get a 9 dimensional system of linear equations,

[3] An excellent survey on this has recently been published by Benhabib and Farmer (1999).

which can be reduced to a 4-dimensional system of difference equations [4] of the form

$$E_t \begin{pmatrix} \hat{C}_{t+1} \\ \hat{K}_{t+1} \\ \hat{Q}_{t+1} \\ \hat{Z}_{t+1} \end{pmatrix} = J \begin{pmatrix} \hat{C}_t \\ \hat{K}_t \\ \hat{Q}_t \\ \hat{Z}_t \end{pmatrix} \tag{10}$$

where J is a (4x4) matrix and $\hat{x}_t = \frac{x_t - x^\circ}{x^\circ}$ and denotes the percentage deviation of x from its steady-state value x°.As state variables we have chosen the aggregate consumption of worker households C_t, the capital stock of worker households K_t, the price of capital Q_t and the capital stock of the entrepreneurs Z_t. The local stability properties of the model can by determined by examining the eigenvalues of the matrix J characterizing this system and in this model, indeterminacy occurs if there are more than two eigenvalues inside the unit circle, since there are two predetermined variables, K_t and Z_t. The expectations operator on the LHS is very important, because only in the case of determinate equilibrium these expectations are uniquely determined by the initial conditions on the predetermined variables. With an indeterminate equilibrium, we must write the resulting system as

$$\begin{pmatrix} \hat{C}_{t+1} \\ \hat{K}_{t+1} \\ \hat{Q}_{t+1} \\ \hat{Z}_{t+1} \end{pmatrix} = J. \begin{pmatrix} \hat{C}_t \\ \hat{K}_t \\ \hat{Q}_t \\ \hat{Z}_t \end{pmatrix} - \begin{pmatrix} E_t(\hat{C}_{t+1}) - \hat{C}_{t+1} \\ 0 \\ E_t(\hat{Q}_{t+1}) - \hat{Q}_{t+1} \\ 0 \end{pmatrix} \tag{11}$$

so that expectational errors can affect the actual values of the free variables. To simulate the model, the matrix J is diagonalized and the effect of the unstable roots of J on the system is excluded. As a consequence of excluding these variables the expectational shocks to the free variables become functions of fundamental innovations or are required to be zero (if there are no fundamental innovations in the system, as is the case in 11. If there are not enough unstable roots, we cannot exclude enough variables and the expectational errors cannot be restricted to zero or functions of fundamental innovations. Expectational errors then lead to fluctuations in real variables.

In this model, the dimension of J is relatively large and the entries of J are each complex functions of the underlying parameters and functions. Therefore we cannot analytically determine the conditions under which indeterminacy arises. Rather we have to analyze numerically the set of plausible calibrations to determine whether and under which conditions the model's equilibrium is indeterminate. The calibration procedure in total requires the

[4] The log-linearized version of the equations can be found in the appendix.

choice of 3 functional forms and 6 parameters. The functional forms need to be chosen for the production function of consumption goods $F(K, H, HE)$, the distribution of the idiosyncratic productivity shock to the capital production technology $\Phi(\omega)$ and the utility function of the representative worker household $U(c_t, 1-h_t)$. The parameters to be calibrated are the share of entrepreneurs in the population η, the shares of capital and entrepreneurial skills in the production of the consumption good α and Γ, the fraction of capital goods destroyed by declaring bankruptcy μ , the discount rates of worker and entrepreneur households β and $\beta\gamma$, and finally the depreciation rate of capital δ.

A plausible choice of the technology parameters requires them to be in line with empirical estimates of corresponding measures. Therefore, we do not assume strong externalities in the production function, but instead assume a Cobb-Douglas production function with constant returns to scale, setting the share of capital to $\alpha = 0.36$. This choice is standard in the macro literature, but most models with perfect financial markets, require some degree of increasing returns to obtain an indeterminate equilibrium. Since the assumption that entrepreneurial skills are used in the production of the consumption good is made only to make the asymmetric information problem well-defined, we set the elasticity parameter to be very small $\Gamma = 0.0001$, which yields a production function for consumption goods of the form

$$Y_t = K_t^{0.36} H_t^{0.6399} HE_t^{0.0001}$$

The model is calibrated with a horizon of one quarter for each period and so we choose the depreciation rate to equal $\delta = 0.02$. The fraction of entrepreneurs in the population is set to $\eta = 0.1$ (close to the share reported by Gentry and Hubbard (2000)) and the fraction of capital goods destroyed during monitoring is set to $\mu = 0.25$. This seems to be a reasonable value, given that liquidation is costly. The distribution of the idiosyncratic productivity shock is assumed to be log-normal, with cumulative distribution function $\Phi(\omega) = \frac{1}{2}\left(1 + Erf[\frac{\sigma^2 + 2\ln[\omega]}{2\sqrt{2}\sigma}\right)$, which only depends on the parameter σ since the mean of the distribution must be 1. Following Carlstrom and Fuerst (1997), we choose to set the standard-deviation σ of the distribution to $\sigma = 0.2$, in order to arrive at an empirically plausible bankruptcy rate of 1%, which is given by $\Phi(\omega^\circ)$ in the model. In the sensitivity analysis, it turned out that altering the choice of any of these parameters except Γ, does not significantly change the results of the simulation. The structure of the resulting system of difference equations is stable with respect to small variations in these parameters. Increasing the weight of entrepreneurial skills in the production of the consumption good Γ leads to strong interactions between worker's consumption and labour supply choices and the net worth of entrepreneurs, potentially yielding unstable or indeterminate equilibria.

With the small value of Γ chosen here, no such problems arise however.

What remains to be chosen at this point is the utility function of the representative household and the discount rates of the agents. Since the time horizon of the period is one quarter, we set $\beta = 0.99$, to arrive at a real interest rate of approximately 4% per year. The discount rate of entrepreneurs is then given by the requirement that the spread exactly offsets the higher internal rate of return on entrepreneurial capital, which implies a value of 0.947 for γ. We allow for a lot of flexibility in the calibration of the utility function of the representative worker household. Without giving a justification, Carlstrom and Fuerst (1997) choose a utility function that is additively separable in consumption and labour, with the coefficient of relative risk aversions set to 1 and the utility function linear in labour with a steady state labour supply of 1/3 of the time endowment. We retain their choice of $h^{\circ} = 0.33$ as the steady-state labour supply, but allow for the full class of preferences concave in c_t and $1 - h_t$.

Rather than restricting the utility function to some specific functional form, we use a set of parameters to identify properties of utility. After log-linearization, the only parameters of the utility function, which appear in the equilibrium conditions are the "second-order" elasticities of the function, evaluated at the steady-state. Since local determinacy of equilibrium can be verified also by checking the linearized version of the equilibrium conditions, we can conveniently work with these four parameters, which are defined as

$$\begin{array}{ll} \epsilon_{cc} = \frac{c^{\circ}}{U_c(c^{\circ},1-h^{\circ})} U_{cc}(c^{\circ}, 1-h^{\circ}) & \epsilon_{ch} = \frac{1-h^{\circ}}{U_c(c^{\circ},1-h^{\circ})} U_{ch}(c^{\circ}, 1-h^{\circ}) \\ \epsilon_{hc} = \frac{c^{\circ}}{U_h(c^{\circ},1-h^{\circ})} U_{hc}(c^{\circ}, 1-h^{\circ}) & \epsilon_{hh} = \frac{1-h^{\circ}}{U_h(c^{\circ},1-h^{\circ})} U_{hh}(c^{\circ}, 1-h^{\circ}) \end{array}$$

where U_c denotes the partial derivative of $U(.)$ with respect to c and U_h denotes the partial derivative of $U(.)$ with respect to $1 - h$. There is an important restriction on the "second-order" cross-elasticities and the "first-order" elasticities, which is derived from the symmetry of the Hessian of $U(.)$. It is given by $\epsilon_{ch} = \frac{\epsilon_h}{\epsilon_c}\epsilon_{hc}$ and can be expressed as $\epsilon_{ch} = \frac{w^{\circ}(1-h^{\circ})}{c^{\circ}}\epsilon_{hc}$ by using the optimal labour supply condition. In order to make sure that the resulting equilibrium is a utility maximum, the instantaneous utility function must also be concave. The concavity conditions require $\epsilon_{cc} < 0$, $\epsilon_{hh} < 0$ and $\epsilon_{hh}\epsilon_{cc} \geq \epsilon_{ch}\epsilon_{hc}$ (see Hintermaier (2003)). Obeying all of these restrictions, three parameter of the utility function remain to be chosen. These are the elasticity of marginal utility from consumption with respect to changes in consumption ϵ_{cc}, the elasticity of marginal utility from leisure with respect to changes in leisure ϵ_{hh} and one of the the cross elasticities

ϵ_{ch}. The concavity condition links these three parameters and the resulting inequality must be fulfilled.

We investigate the stability of the saddle-path equilibrium by calculating the eigenvalues of J for 100.000 random draws from the parameter space $\Omega = \{\epsilon_{cc}, \epsilon_{ch}, \epsilon_{hh}\}$ with the restrictions $\epsilon_{cc} \in]-100, 0]$ and $\epsilon_{ch}]-100, 100[$ and $\epsilon_{hh} \leq \frac{\epsilon_{ch}\epsilon_{hc}}{\epsilon_{cc}}$. The upper bound of the first interval is given by the restriction that $\epsilon_{cc} < 0$ from the concavity condition and the lower bound represents a relatively arbitrarily chosen extreme value (for separable utility functions, $-\epsilon_{cc}$ equals the coefficient of risk aversion). The bounds of the second interval are chosen to be 100 for reasons of symmetry and computational efficiency. The restriction on ϵ_{hh} is sufficient to ensure concavity, since the right hand side of the relevant inequality is always negative. The result of our computations are conveniently summarized in Figure 1.

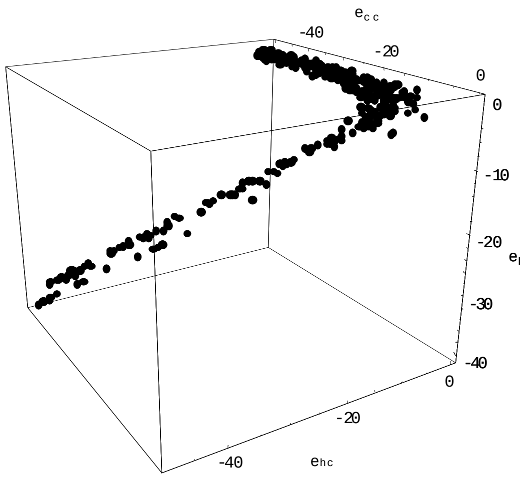


Figure 1: Indeterminacy region in parameter space

The graph shows the location of parameter combinations yielding an indeterminate equilibrium of the economy in the 3-dimensional parameter space described above. The concavity restriction is not depicted in the graph, but the points all lie close to its surface. The fact that parameter combinations are close to the surface implied by the concavity restriction implies that the utility function cannot be too concave in order for the equilibrium of the economy to be indeterminate. Another structural feature of the indeterminate equilibria is that the first eigenvalue of J is always unstable. Hence, it is not the consumption shock that drives the fluctuations of the economy, but rather the shock to the expected price of capital

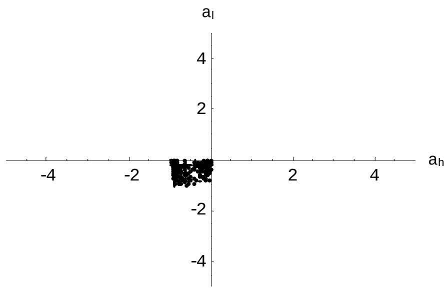


Figure 2: Indeterminacy region in space of Frisch labour supply curve coefficients

$E_t(\hat{Q}_{t+1}) - \hat{Q}_{t+1}$. This is in contrast to other models of indeterminate equilibrium, which have relied on shocks to expected consumption instead.

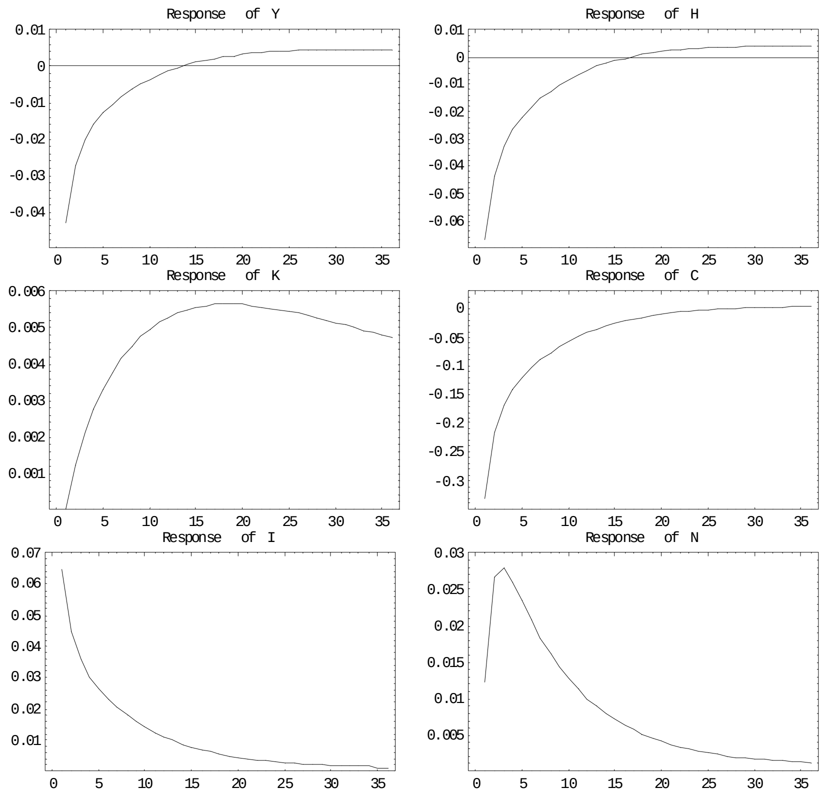
That the slopes of labour supply curves play an important role for the stability properties of the equilibrium in dynamic general equilibrium models is well known (see Benhabib and Farmer (1999)). We now illustrate the properties of the labour market needed to obtain an indeterminate equilibrium in order to understand better what is required from the utility function to generate indeterminacy. We have computed the implied coefficients for the Frisch labour supply curve

$$\hat{w} = const + \alpha_h \hat{h} + \alpha_\lambda \hat{\lambda}$$

where $\alpha_h = \epsilon_{hh} - \frac{\epsilon_{ch}\epsilon_{hc}}{\epsilon_{cc}}$ and $\alpha_\lambda = \frac{\epsilon_{hc}}{\epsilon_{cc}} - 1$. Figure 2 shows the combinations of Frisch labour supply curve coefficients computed from the parameter sets generating indeterminacy. The Frisch labour supply curve is derived by holding constant the marginal utility of consumption and as one can see both coefficients must be negative and smaller than 1 in absolute size. This means that the labour supply curve must be downward-sloping, but very flat and stable with respect to changes in the marginal utility of wealth. Small changes in the real wage have a strong effect on hours worked and hence output will be volatile in response to changes in the capital stock.

Another way to study the mechanism that gives rise to indeterminacy is to look at the impulse response functions. Below we plot the impulse response function for total consumption (C), total capital (K), the price of capital (Q), the monitoring threshold (w), entrepreneur's capital stock (Z), entrepreneur's net worth (N), total investment spending (I) and total output (Y) in response to a positive 1% shock to the expected price of capital. All 9 graphs have the same format with the vertical axis measuring the percentage deviation from the steady state and the horizontal axis measuring the number of quarters passed after the initial shock.

Figure 3: Impulse response functions



If the agents expect a higher price of capital in the future they invest more and consume less. While the rise in the capital stock is contained by an increase in monitoring costs, employment falls with consumption, and as a result of this, also output falls, but by not as much as consumption. The price of capital increases because of the higher demand, which raises the future net worth of entrepreneurs. In the following periods investment remains high because of the persistent net worth increase but the price of capital quickly falls back to its initial level. This reduces entrepreneurial net worth, and despite the adjustment of the threshold level of monitoring, as investment slowly decreases also the rise in the capital stock is slowing. The capital stock peaks after about 16 quarters and for some time consumption, hours, and output actually rise above their steady-state values pulling the total capital stock towards its steady-state value over the long term.

4 Conclusion

We have shown that in a 1-sector SDGE-model of the business cycle with imperfect financial markets, the equilibrium paths of real variables might be locally indeterminate even if the aggregate production function for output has constant returns to scale. This result is interesting in itself because in the standard perfect financial markets case, some degree of increasing returns to scale is necessary in order to get locally indeterminate equilibria (see Hintermaier (2003)). The mechanics of the model with financial market imperfections are therefore closer to those of a 2-sector model for which it is well known that even with aggregate constant returns to scale models with locally indeterminate equilibria exist.

Another interesting innovation of the financial market imperfections model is that the source of business cycle fluctuations are not shocks to consumer's expectations about future consumption, but shocks to entrepreneur's expectations of the future price of capital which are then transmitted via labor demand to consumption and output. This type of shock corresponds more closely to the "animal spirits" that Keynes and his followers focused on in their explanation of business cycles. It might also be more relevant for recent developments in the industrialized economies when a surging stock market coincided with strong real output and consumption growth and a recession followed a steep drop in major stock market indices. While the model presented cannot account for the observed correlation between developments in the stock market and real output growth yet, we have laid out a framework in which an empirical analysis of this issue can be successful.

Future research has to be directed towards studying more closely the type of financial market imperfection that gives rise to local indeterminacy and comovement between stock prices and real output growth. The main empirical issue that needs to be resolved is the lack of positive correlation between investment and consumption which is a consequence of the elimination of technology shocks as a source of output variation. A model with countercyclical agency costs should be very well suited to overcome this type of problem.

5 Appendix

The log-linearized system

$$(-\epsilon_{cc}+\epsilon_{hc})\,\hat{C}_t=(-\alpha-\Gamma-\epsilon_{ch}+\epsilon_{hh})\,\hat{H}_t+\alpha\hat{K}_t \tag{1*}$$

$$\epsilon_{cc}\hat{C}_t-\epsilon_{ch}\hat{H}_t+\hat{Q}_t-\epsilon_{cc}\hat{C}_{t+1}= \tag{2*}$$
$$=(\frac{R^\circ\,(1-\alpha-\Gamma)}{R^\circ+Q^\circ\,(1-\delta)}-\epsilon_{ch})\hat{H}_{t+1}+\frac{R^\circ\,(-1+\alpha)}{R^\circ+Q^\circ\,(1-\delta)}\hat{K}_{t+1}+\frac{Q^\circ\,(1-\delta)}{R^\circ+Q^\circ\,(1-\delta)}\hat{Q}_{t+1}$$

$$\hat{Q}_t=\frac{\mu\omega^\circ(\frac{f(\omega^\circ)\phi(\omega^\circ)^2}{(1-\Phi(\omega^\circ))^2}-\frac{f(\omega^\circ)\phi'(\omega^\circ)}{-1+\Phi(\omega^\circ)})}{Q^\circ}\hat{\omega}_t \tag{3*}$$

$$\hat{I}_t=\hat{N}_t+\frac{Q^\circ g(\omega^\circ)}{1-Q^\circ g(\omega^\circ)}\hat{Q}_t+\frac{N^\circ Q^\circ\omega^\circ(1-\mu\phi(\omega^\circ)-\Phi(\omega^\circ))}{I^\circ}\hat{\omega}_t \tag{4*}$$

$$(\eta X^\circ(-1+\alpha+\Gamma)-R^\circ(1-\alpha-\Gamma)Z^\circ)\hat{H}_t+(-\eta X^\circ\alpha-R^\circ(-1+\alpha)Z^\circ)\hat{K}_t+N^\circ\hat{N}_t= \tag{5*}$$
$$=Q^\circ(1-\delta)Z^\circ\hat{Q}_t+(N^\circ-\eta X^\circ)\hat{Z}_t$$

$$\hat{Q}_t-(\frac{Q^\circ\,(1-\delta)}{R^\circ+Q^\circ\,(1-\delta)}+\frac{1}{1-Q^\circ g(\omega^\circ)})\hat{Q}_{t+1}= \tag{6*}$$
$$=\frac{R^\circ\,(1-\alpha-\Gamma)}{R^\circ+Q^\circ\,(1-\delta)}\hat{H}_{t+1}+\frac{R^\circ\,(-1+\alpha)}{R^\circ+Q^\circ\,(1-\delta)}\hat{K}_{t+1}+(\frac{\omega^\circ f'(\omega^\circ)}{f(\omega^\circ)}+\frac{\omega^\circ Q^\circ g'(\omega^\circ)}{1-Q^\circ g(\omega^\circ)})\hat{\omega}_{t+1}$$

$$\frac{CE^\circ}{Q^\circ Z^\circ+CE^\circ}C\hat{E}_t+\frac{Q^\circ Z^\circ}{Q^\circ Z^\circ+CE^\circ}\hat{Z}_{t+1}= \tag{7*}$$
$$\hat{N}_t+(\frac{CE^\circ}{Q^\circ Z^\circ+CE^\circ}+\frac{Q^\circ g(\omega^\circ)}{1-Q^\circ g(\omega^\circ)})\hat{Q}_t+(\frac{\omega^\circ f'(\omega^\circ)}{f(\omega^\circ)}+\frac{\omega^\circ Q^\circ g'(\omega^\circ)}{1-Q^\circ g(\omega^\circ)})\hat{\omega}_t$$

$$(1-\alpha-\Gamma)\,\hat{H}_t+\alpha\hat{K}_t=\frac{C^\circ}{Y^\circ}\hat{C}_t+\frac{CE^\circ}{Y^\circ}C\hat{E}_t+\frac{I^\circ}{Y^\circ}\hat{I}_t \tag{8*}$$

$$-\frac{(1-\delta)}{\delta}\hat{K}_t+\frac{1}{\delta}\hat{K}_{t+1}=\hat{I}_t+\frac{\mu\omega^\circ\phi(\omega^\circ)}{1-\mu\Phi(\omega^\circ)}\hat{\omega}_t \tag{9*}$$

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